◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

## INCOHERENCE CORRECTION STRATEGIES IN STATISTICAL MATCHING

#### Andrea Capotorti and Barbara Vantaggi

#### 26-VII-2011

## About ourselves...

## Both of us come from the so called ... "0-probability Italian school" !!!

and in particular:

- Barbara is Associate Professor at Roma "la Sapienza";
- I'm University-Researcher (Assistant Professor) in Perugia;

the "Rome-Perugia" (Coletti& Scozzafava) Axis !

▲ロト ▲帰ト ▲ヨト ▲ヨト 三日 - の々ぐ

## About ourselves...

## Both of us come from the so called ... "0-probability Italian school" !!!

and in particular:

- Barbara is Associate Professor at Roma "la Sapienza";
- I'm University-Researcher (Assistant Professor) in Perugia;

the "Rome-Perugia" (Coletti& Scozzafava) Axis !

▲ロト ▲帰ト ▲ヨト ▲ヨト 三日 - の々ぐ

## About ourselves...

# Both of us come from the so called ... "0-probability Italian school" !!!

#### and in particular:

- Barbara is Associate Professor at Roma "la Sapienza";
- I'm University-Researcher (Assistant Professor) in Perugia; the "Rome-Perugia" (Coletti& Scozzafava) Axis !

## About ourselves...

Both of us come from the so called ... "0-probability Italian school" !!!

and in particular:

• Barbara is Associate Professor at Roma "la Sapienza";

I'm University-Researcher (Assistant Professor) in Perugia;
 the "Rome-Perugia" (Coletti& Scozzafava) Axis !

## About ourselves...

Both of us come from the so called ... "0-probability Italian school" !!!

and in particular:

- Barbara is Associate Professor at Roma "la Sapienza";
- I'm University-Researcher (Assistant Professor) in Perugia;

the "Rome-Perugia" (Coletti& Scozzafava) Axis !

```
About ourselves...
```

Both of us come from the so called ... "0-probability Italian school" !!!

and in particular:

- Barbara is Associate Professor at Roma "la Sapienza";
- I'm University-Researcher (Assistant Professor) in Perugia; the "Rome-Perugia" (Coletti& Scozzafava) Axis !

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ ○臣○

## OUR "CREDO"

### Probability doesn't exists ! (de Finetti's provocative statement)

#### +

Probabilistic evaluations are commonly conditional and partial (our "mantra")

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ ○臣○

## OUR "CREDO"

## Probability doesn't exists ! (de Finetti's provocative statement)

## Probabilistic evaluations are commonly conditional and partial (our "mantra")

◆□▶ ◆□▶ ◆□▶ ◆□▶ ●□

## OUR "CREDO"

## Probability doesn't exists ! (de Finetti's provocative statement)

## Probabilistic evaluations are commonly conditional and partial (our "mantra")

## OUR "CREDO"

## Probability doesn't exists ! (de Finetti's provocative statement)

+

## Probabilistic evaluations are commonly conditional and pa (our "mantra")

## OUR "CREDO"

### Probability doesn't exists ! (de Finetti's provocative statement)

#### +

Probabilistic evaluations are commonly conditional and partial (our "mantra")

## OUR "CREDO"

### Probability doesn't exists ! (de Finetti's provocative statement)

#### +

Probabilistic evaluations are commonly conditional and partial (our "mantra")

## OUR "CREDO"

### Probability doesn't exists ! (de Finetti's provocative statement)

#### +

Probabilistic evaluations are commonly conditional and partial (our "mantra")

## OUR "CREDO"

Probability doesn't exists ! (de Finetti's provocative statement)

#### +

Probabilistic evaluations are commonly conditional and partial (our "mantra")

## OUR "CREDO"

Probability doesn't exists ! (de Finetti's provocative statement)

#### +

Probabilistic evaluations are commonly conditional and partial (our "mantra")

## OUR CONTRIBUTION

## The present paper comes from the merging (hence it's (*meta*)matching [:-)) of two our previous ISIPTA's contribution

- B. Vantaggi. The role of coherence for the integration of different sources. *ISIPTA*'05 (Pittsburgh, USA):
- A. Capotorti, G. Regoli and F. Vattari. On the use of a new discrepancy measure to correct incoherent assessments and to aggregate conflicting opinions based on imprecise conditional probabilities. *ISIPTA*'09 - Durham (UK).

## OUR CONTRIBUTION

#### The present paper comes from the merging (hence it's

(*meta*)matching ! ;-) ) of two our previous ISIPTA's contributions:

- B. Vantaggi. The role of coherence for the integration of different sources. *ISIPTA*'05 (Pittsburgh, USA):
- A. Capotorti, G. Regoli and F. Vattari. On the use of a new discrepancy measure to correct incoherent assessments and to aggregate conflicting opinions based on imprecise conditional probabilities. *ISIPTA*'09 - Durham (UK).

## OUR CONTRIBUTION

## The present paper comes from the merging (hence it's (*meta*)matching ! ;-)) of two our previous ISIPTA's contributions:

- B. Vantaggi. The role of coherence for the integration of different sources. *ISIPTA*'05 (Pittsburgh, USA):
- A. Capotorti, G. Regoli and F. Vattari. On the use of a new discrepancy measure to correct incoherent assessments and to aggregate conflicting opinions based on imprecise conditional probabilities. *ISIPTA*'09 - Durham (UK).

## OUR CONTRIBUTION

## The present paper comes from the merging (hence it's (*meta*)matching ! ;-) ) of two our previous ISIPTA's contributions:

- B. Vantaggi. The role of coherence for the integration of different sources. *ISIPTA*'05 (Pittsburgh, USA):
- A. Capotorti, G. Regoli and F. Vattari. On the use of a new discrepancy measure to correct incoherent assessments and to aggregate conflicting opinions based on imprecise conditional probabilities. *ISIPTA*'09 - Durham (UK).

## OUR CONTRIBUTION

The present paper comes from the merging (hence it's (*meta*)matching ! ;-) ) of two our previous ISIPTA's contributions:

- B. Vantaggi. The role of coherence for the integration of different sources. *ISIPTA*'05 (Pittsburgh, USA):
- A. Capotorti, G. Regoli and F. Vattari. On the use of a new discrepancy measure to correct incoherent assessments and to aggregate conflicting opinions based on imprecise conditional probabilities. *ISIPTA*'09 - Durham (UK).

- we deal with the managing of inconsistencies inside the Statistical Matching (integration of sources) framework;
  - when logical relations among the variables are present incoherence can arise in the probability evaluations
- different methods can be used to remove such incoherences:
  - maximize the "partial likelihood function" on the base of observed data;
  - least committal imprecise probability extensions;
  - specific precise "distances" minimization.

- we deal with the managing of inconsistencies inside the Statistical Matching (integration of sources) framework;
  - when logical relations among the variables are present incoherence can arise in the probability evaluations
- different methods can be used to remove such incoherences:
  - maximize the "partial likelihood function" on the base of observed data;
  - least committal imprecise probability extensions;
  - specific precise "distances" minimization.

- we deal with the managing of inconsistencies inside the Statistical Matching (integration of sources) framework;
  - when logical relations among the variables are present incoherence can arise in the probability evaluations
- different methods can be used to remove such incoherences:
  - maximize the "partial likelihood function" on the base of observed data;
  - least committal imprecise probability extensions;
  - specific precise "distances" minimization.

- we deal with the managing of inconsistencies inside the Statistical Matching (integration of sources) framework;
  - when logical relations among the variables are present incoherence can arise in the probability evaluations
- different methods can be used to remove such incoherences:
  - maximize the "partial likelihood function" on the base of observed data;
  - least committal imprecise probability extensions;
  - specific precise "distances" minimization.

- we deal with the managing of inconsistencies inside the Statistical Matching (integration of sources) framework;
  - when logical relations among the variables are present incoherence can arise in the probability evaluations
- different methods can be used to remove such incoherences:
  - maximize the "partial likelihood function" on the base of observed data;
  - least committal imprecise probability extensions;
  - specific precise "distances" minimization.

## COHERENT EXTENSION

To adjust the initially incoherent assessment  $(\mathcal{E}, \mathbf{p})$  it is possible to determine a coherent sub-assessment  $(\mathcal{G}, \mathbf{p}_{|\mathcal{G}})$  with maximal cardinality and coherently extend it to the rest  $\mathcal{F} = \mathcal{E} \setminus \mathcal{G}$  by the generalized Bayesian updating scheme obtaining an imprecise sub-assessment

 $(\mathcal{F}, [\underline{p_{\mathcal{F}}}, \overline{p_{\mathcal{F}}}]).$ 

Note that inference on decision targets can be performed again through the generalized Bayesian updating scheme but applied to imprecise evaluations.

Whenever too vague, inference bounds can be eventually reduced to coherent cores, i.e. *total* coherent subintervals with highest degree of support.

## COHERENT EXTENSION

To adjust the initially incoherent assessment  $(\mathcal{E}, \mathbf{p})$  it is possible to determine a coherent sub-assessment  $(\mathcal{G}, \mathbf{p}_{|\mathcal{G}})$  with maximal cardinality and coherently extend it to the rest  $\mathcal{F} = \mathcal{E} \setminus \mathcal{G}$  by the generalized Bayesian updating scheme obtaining an imprecise sub-assessment

 $(\mathcal{F}, [\underline{p_{\mathcal{F}}}, \overline{p_{\mathcal{F}}}]).$ 

Note that inference on decision targets can be performed again through the generalized Bayesian updating scheme but applied to imprecise evaluations.

Whenever too vague, inference bounds can be eventually reduced to coherent cores, i.e. *total* coherent subintervals with highest degree of support.

## COHERENT EXTENSION

To adjust the initially incoherent assessment  $(\mathcal{E}, \mathbf{p})$  it is possible to determine a coherent sub-assessment  $(\mathcal{G}, \mathbf{p}_{|\mathcal{G}})$  with maximal cardinality and coherently extend it to the rest  $\mathcal{F} = \mathcal{E} \setminus \mathcal{G}$  by the generalized Bayesian updating scheme obtaining an imprecise sub-assessment

 $(\mathcal{F}, [\underline{p_{\mathcal{F}}}, \overline{p_{\mathcal{F}}}]).$ 

Note that inference on decision targets can be performed again through the generalized Bayesian updating scheme but applied to imprecise evaluations.

Whenever too vague, inference bounds can be eventually reduced to coherent cores, i.e. *total* coherent subintervals with highest degree of support.

## (PSEUDO)DISTANCES AMONG PROBABILITY DISTRIBUTIONS

Given two conditional assessments  $\mathbf{p} = [p_1, \dots, p_n]$  and  $\mathbf{q} = [q_1, \dots, q_n]$  on the same set of conditional events  $\mathcal{E} = [E_1|H_1, \dots, E_n|H_n]$ , the most widely adopted divergencies among them are:

L1(p,q) = 
$$\sum_{i=1}^{n} |q_i - p_i|$$
;
L2(p,q) =  $\sum_{i=1}^{n} (q_i - p_i)^2$ ;
KL(p,q) =  $\sum_{i=1}^{n} (q_i \ln(q_i/p_i) - q_i + p_i)$ .

## (PSEUDO)DISTANCES AMONG PROBABILITY DISTRIBUTIONS

Given two conditional assessments  $\mathbf{p} = [p_1, \dots, p_n]$  and  $\mathbf{q} = [q_1, \dots, q_n]$  on the same set of conditional events  $\mathcal{E} = [E_1|H_1, \dots, E_n|H_n]$ , the most widely adopted divergencies among them are:

L1(p,q) = 
$$\sum_{i=1}^{n} |q_i - p_i|$$
;
L2(p,q) =  $\sum_{i=1}^{n} (q_i - p_i)^2$ ;
KL(p,q) =  $\sum_{i=1}^{n} (q_i \ln(q_i/p_i) - q_i + p_i)$ .

## OUR MAIN TOOL

... but for partial conditional probability assessments  ${\bf p}\in (0,1)^n$  on  ${\cal E}$  recently we tailored the following "discrepancy"

$$\Delta(\mathbf{p}, oldsymbol{lpha}) = \sum_{i \mid lpha(H_i) > 0} lpha(H_i) \left( q_i \ln rac{q_i}{p_i} + (1 - q_i) \ln rac{(1 - q_i)}{(1 - p_i)} 
ight),$$

## where $\mathbf{q}_{\pmb{\alpha}}$ is an assessment on $\mathcal E$ induced by the probability mass distribution $\pmb{\alpha},$

that now we specialize for Statistical Matching with a mixture of discrepancies  $\Delta_{mix}(\mathbf{p}, \{\alpha_i\}_i)...$ 

## OUR MAIN TOOL

... but for partial conditional probability assessments  ${\bf p}\in (0,1)^n$  on  ${\cal E}$  recently we tailored the following "discrepancy"

$$\Delta(\mathbf{p}, \boldsymbol{\alpha}) = \sum_{i \mid \alpha(H_i) > 0} \alpha(H_i) \left( q_i \ln \frac{q_i}{p_i} + (1 - q_i) \ln \frac{(1 - q_i)}{(1 - p_i)} \right),$$

where  $\mathbf{q}_{\alpha}$  is an assessment on  $\mathcal E$  induced by the probability mass distribution  $\alpha$ ,

that now we specialize for Statistical Matching with a mixture of discrepancies  $\Delta_{mix}(\mathbf{p}, \{\alpha_i\}_i)...$ 

INTEGRATION OF SOURCES IN A COHERENT SETTING

 $(X_1, Y_1), ..., (X_{n_A}, Y_{n_A})$  and  $(X_{n_A+1}, Z_{n_A+1}), ..., (X_{n_A+n_B}, Z_{n_A+n_B})$ two random samples (with a finite range) related to two sources Aand B concerning the same population of interest, and drawn according to the same sampling scheme, with

 $(X_1, Y_1), ..., (X_{n_A}, Y_{n_A})$  and  $(X_{n_A+1}, Z_{n_A+1}), ..., (X_{n_A+n_B}, Z_{n_A+n_B})$  exchangeable, as well as the sequence

 $X_1, ..., X_{n_A}, X_{n_A+1}, ..., X_{n_A+n_B}$ 

We can elicit from the two files the relevant probability values:

$$y_{j|i} = P_{Y|(X=x_i)}(Y=y_j) \, z_{k|i} = P_{Z|(X=x_i)}(Z=z_k) \, x_i = P_X(X=x_i)$$

INTEGRATION OF SOURCES IN A COHERENT SETTING

 $(X_1, Y_1), ..., (X_{n_A}, Y_{n_A})$  and  $(X_{n_A+1}, Z_{n_A+1}), ..., (X_{n_A+n_B}, Z_{n_A+n_B})$ two random samples (with a finite range) related to two sources Aand B concerning the same population of interest, and drawn according to the same sampling scheme, with

 $(X_1, Y_1), ..., (X_{n_A}, Y_{n_A})$  and  $(X_{n_A+1}, Z_{n_A+1}), ..., (X_{n_A+n_B}, Z_{n_A+n_B})$  exchangeable, as well as the sequence

 $X_1, ..., X_{n_A}, X_{n_A+1}, ..., X_{n_A+n_B}$ 

We can elicit from the two files the relevant probability values:

$$y_{j|i} = P_{Y|(X=x_i)}(Y=y_j) z_{k|i} = P_{Z|(X=x_i)}(Z=z_k) x_i = P_X(X=x_i)$$

INTEGRATION OF SOURCES IN A COHERENT SETTING

 $(X_1, Y_1), ..., (X_{n_A}, Y_{n_A})$  and  $(X_{n_A+1}, Z_{n_A+1}), ..., (X_{n_A+n_B}, Z_{n_A+n_B})$ two random samples (with a finite range) related to two sources Aand B concerning the same population of interest, and drawn according to the same sampling scheme, with

 $(X_1, Y_1), ..., (X_{n_A}, Y_{n_A})$  and  $(X_{n_A+1}, Z_{n_A+1}), ..., (X_{n_A+n_B}, Z_{n_A+n_B})$  exchangeable, as well as the sequence

 $X_1, ..., X_{n_A}, X_{n_A+1}, ..., X_{n_A+n_B}$ 

We can elicit from the two files the relevant probability values:

$$y_{j|i} = P_{Y|(X=x_i)}(Y=y_j) \, z_{k|i} = P_{Z|(X=x_i)}(Z=z_k) \, x_i = P_X(X=x_i)$$

INTEGRATION OF SOURCES IN A COHERENT SETTING

 $(X_1, Y_1), ..., (X_{n_A}, Y_{n_A})$  and  $(X_{n_A+1}, Z_{n_A+1}), ..., (X_{n_A+n_B}, Z_{n_A+n_B})$ two random samples (with a finite range) related to two sources Aand B concerning the same population of interest, and drawn according to the same sampling scheme, with

 $(X_1, Y_1), ..., (X_{n_A}, Y_{n_A})$  and  $(X_{n_A+1}, Z_{n_A+1}), ..., (X_{n_A+n_B}, Z_{n_A+n_B})$  exchangeable, as well as the sequence

$$X_1, ..., X_{n_A}, X_{n_A+1}, ..., X_{n_A+n_B}$$

We can elicit from the two files the relevant probability values:

$$y_{j|i} = P_{Y|(X=x_i)}(Y=y_j) z_{k|i} = P_{Z|(X=x_i)}(Z=z_k) x_i = P_X(X=x_i)$$

#### ... AND THE MAIN PROBLEM

Given  $y_{j|i}, z_{k|i}, x_i$ , for any i, j, k, with  $(y_{j|i}, x_i)$  and  $(z_{k|i}, x_i)$ separately coherent and with some logical constraints among the variables Y and Z, incoherence in the **whole assessment** 

$$(\mathcal{E}, \mathbf{p}) \text{ with } \begin{array}{l} \mathcal{E} = \left\{ \begin{array}{l} (X = x_i), \ (Y = y_j) | (X = x_i), \ (Z = z_k) | (X = x_i) \\ \text{ for any } x_i, y_j, z_k \end{array} \right\}, \\ \mathbf{p} = \{x_i, y_{j|i}, z_{k|i}\}_{i,j,k} \quad . \end{array}$$

can *localize* only in association to elements of  $\mathcal{E}$  with the same conditioning event  $(X = x_i)$ .

#### A MIXTURE OF DISCREPANCIES

Hence our discrepancy  $\Delta(\mathbf{p}, \alpha)$  can be reformulated into

$$egin{aligned} \Delta_{mix}(\mathbf{p}, \{m{lpha}_i\}_i) &= \sum_i arkappa_i \left[ \sum_j \left( q_{j|i}^{lpha_i} \ln rac{q_{j|i}^{lpha_i}}{arphi_{j|i}} + (1 - q_{j|i}^{lpha_i}) \ln rac{(1 - q_{j|i}^{lpha_i})}{(1 - arphi_{j|i})} 
ight) + \ &+ \sum_k \left( q_{k|i}^{lpha_i} \ln rac{q_{k|i}^{lpha_i}}{arkappa_{k|i}} + (1 - q_{k|i}^{lpha_i}) \ln rac{(1 - q_{k|i}^{lpha_i})}{(1 - arkappa_{k|i})} 
ight) 
ight] \end{aligned}$$

where each distribution  $\alpha_i$  works just on the sample space spanned by the conditional events  $\{(Y = y_j) | (X = x_i), (Z = z_k) | (X = x_i)\}$ , it is constrained to fulfill the normalizing condition  $\alpha_i(X = x_i) = x_i$ , and generates the conditional probabilities

$$q_{j|i}^{\alpha_i} = \frac{\alpha_i(Y = y_j)}{\alpha_i(X = x_i)} \quad q_{k|i}^{\alpha_i} = \frac{\alpha_i(Z = z_k)}{\alpha_i(X = x_i)}.$$

#### OUR RESULTS

We applied the three methodologies (likelihood maximization, coherent extension and distances minimizations) to real data representing a subset of employees extracted from a pilot survey of the Italian Population and Household Census that induce an incoherent initial assessment.

Three categorical variables have been analyzed: Age, Educational Level and Professional Status.

And we obtained...

р				
			0.2914	

... details at the poster ! but here we can say...

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ●

EXAMPLE

#### OUR RESULTS

We applied the three methodologies (likelihood maximization, coherent extension and distances minimizations) to real data representing a subset of employees extracted from a pilot survey of the Italian Population and Household Census that induce an incoherent initial assessment.

Three categorical variables have been analyzed: Age, Educational Level and Professional Status.

And we obtained...  $5_1|A_4$   $5_2|A_4$   $5_3|A_4$ P 0.6667 0.1111 0.2222 P 0.6667 0.227

	$S_1   A_4$	$S_2 A_4$	$S_3 A_4$	$E_1 A_4$	$E_2   A_4$	$E_3 A_4$	$E_4   A_4$	$ S_3 E_4$
р	0.6667	0.1111	0.2222	0.6667	0	0.2000	0.1333	Ø
$L1_{ \mathcal{F}}$	0.2222	-	0.6667	0.6667	-	-	-	[0,0.6285]
L1 A4	0.5266	0	0.4734	0.4734	0	0.2836	0.2431	[0,0.6234]
L2 A4	0.5333	0.0389	0.4278	0.4278	0.0389	0.3	0.2333	[0,0.6238]
KĽ <sub>IA4</sub>	0.4856	0.1179	0.3965	0.3965	0.1179	0.2914	0.1942	[0,0.6257]
$\Delta_{mix}$	0.4985	0.0939	0.4077	0.4077	0.0939	0.2943	0.2042	[0,0.6252]
ML	0.4286	0.0714	0.5000	0.5000	0	0.3000	0.2000	[0,0.6254]
$IP_{\mathcal{E}\setminus\mathcal{F}}$	[0, 0.2222]	-	[0.6667 0.8889]	-	-	-	-	[0,0.6386]
core								[0.0017,0.6286]
$IP_{\mathcal{E} \setminus \{\cdot   A_4\}}$	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,1]	[0,0.6607]
core								[0,0.6349]

... details at the poster ! but here we can say...

◆□▶ ◆圖▶ ★ 副▶ ★ 副▶ → 副 → のへで

- conditional events involved in incoherencies are those conditioned to the age status A<sub>4</sub>, in particular, the minimal set is F = {E<sub>1</sub>|A<sub>4</sub>, S<sub>1</sub>|A<sub>4</sub>, S<sub>3</sub>|A<sub>4</sub>};
- L1<sub>|F</sub> and IP<sub>E\F</sub> perform quite well: even though a drastic change on the probability values, they induce quite reasonable inference bounds;
- $L1_{|A4}$  and ML give similar results and in particular they leave to 0 the probability of  $E_2|A_4$  since the absence of observations in the original data;
- others "precise" adjustments have all quite similar behaviors;
- $\Delta_{mix}$  has the advantage of automatically localize of the scenarios where the adjustment can be performed;
- the wider imprecise correction *IP*<sub>E \{:|A4</sub>}, being the one with less assumption requirement, surely performs worst.

- conditional events involved in incoherencies are those conditioned to the age status A<sub>4</sub>, in particular, the minimal set is F = {E<sub>1</sub>|A<sub>4</sub>, S<sub>1</sub>|A<sub>4</sub>, S<sub>3</sub>|A<sub>4</sub>};
- L1<sub>|F</sub> and IP<sub>E\F</sub> perform quite well: even though a drastic change on the probability values, they induce quite reasonable inference bounds;
- $L1_{|A4}$  and ML give similar results and in particular they leave to 0 the probability of  $E_2|A_4$  since the absence of observations in the original data;
- others "precise" adjustments have all quite similar behaviors;
- $\Delta_{mix}$  has the advantage of automatically localize of the scenarios where the adjustment can be performed;
- the wider imprecise correction  $IP_{\mathcal{E}\setminus\{:|A_4\}}$ , being the one with less assumption requirement, surely performs worst.

- conditional events involved in incoherencies are those conditioned to the age status A<sub>4</sub>, in particular, the minimal set is F = {E<sub>1</sub>|A<sub>4</sub>, S<sub>1</sub>|A<sub>4</sub>, S<sub>3</sub>|A<sub>4</sub>};
- L1<sub>|F</sub> and IP<sub>E\F</sub> perform quite well: even though a drastic change on the probability values, they induce quite reasonable inference bounds;
- $L1_{|A4}$  and ML give similar results and in particular they leave to 0 the probability of  $E_2|A_4$  since the absence of observations in the original data;
- others "precise" adjustments have all quite similar behaviors;
- $\Delta_{mix}$  has the advantage of automatically localize of the scenarios where the adjustment can be performed;
- the wider imprecise correction  $IP_{\mathcal{E}\setminus\{:|A_4\}}$ , being the one with less assumption requirement, surely performs worst.

- conditional events involved in incoherencies are those conditioned to the age status A<sub>4</sub>, in particular, the minimal set is F = {E<sub>1</sub>|A<sub>4</sub>, S<sub>1</sub>|A<sub>4</sub>, S<sub>3</sub>|A<sub>4</sub>};
- L1<sub>|F</sub> and IP<sub>E\F</sub> perform quite well: even though a drastic change on the probability values, they induce quite reasonable inference bounds;
- $L1_{|A4}$  and ML give similar results and in particular they leave to 0 the probability of  $E_2|A_4$  since the absence of observations in the original data;
- others "precise" adjustments have all quite similar behaviors;
- $\Delta_{mix}$  has the advantage of automatically localize of the scenarios where the adjustment can be performed;
- the wider imprecise correction  $IP_{\mathcal{E}\setminus\{:|A_4\}}$ , being the one with less assumption requirement, surely performs worst.

- conditional events involved in incoherencies are those conditioned to the age status A<sub>4</sub>, in particular, the minimal set is F = {E<sub>1</sub>|A<sub>4</sub>, S<sub>1</sub>|A<sub>4</sub>, S<sub>3</sub>|A<sub>4</sub>};
- L1<sub>|F</sub> and IP<sub>E\F</sub> perform quite well: even though a drastic change on the probability values, they induce quite reasonable inference bounds;
- $L1_{|A4}$  and ML give similar results and in particular they leave to 0 the probability of  $E_2|A_4$  since the absence of observations in the original data;
- others "precise" adjustments have all quite similar behaviors;
- $\Delta_{mix}$  has the advantage of automatically localize of the scenarios where the adjustment can be performed;
- the wider imprecise correction  $IP_{\mathcal{E}\setminus\{\cdot|A_4\}}$ , being the one with less assumption requirement, surely performs worst.

- conditional events involved in incoherencies are those conditioned to the age status A<sub>4</sub>, in particular, the minimal set is F = {E<sub>1</sub>|A<sub>4</sub>, S<sub>1</sub>|A<sub>4</sub>, S<sub>3</sub>|A<sub>4</sub>};
- L1<sub>|F</sub> and IP<sub>E\F</sub> perform quite well: even though a drastic change on the probability values, they induce quite reasonable inference bounds;
- $L1_{|A4}$  and ML give similar results and in particular they leave to 0 the probability of  $E_2|A_4$  since the absence of observations in the original data;
- others "precise" adjustments have all quite similar behaviors;
- $\Delta_{mix}$  has the advantage of automatically localize of the scenarios where the adjustment can be performed;
- the wider imprecise correction  $IP_{\mathcal{E}\setminus\{\cdot|A_4\}}$ , being the one with less assumption requirement, surely performs worst.

#### A FINAL REMARK...

This research is financed by national ministry PRIN research project ....

... but till now mainly influenced by government financial and motivational destruction of public and free universities





SELF-INTRO

EXAMPLE

#### Rome-Perugia Axis...





◆□ > ◆□ > ◆ □ > ◆ □ > □ = のへで